2019

PHYSICS

(Major)

Paper : 3.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Methods)

(*Marks* : 25)

1. Answer the following questions : 1×2=2

(a) What is idempotent matrix?

(b) Define adjoint of a matrix.

2. For the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$$

verify the theorem

$$A(\text{adj } A) = (\text{adj } A)A = |A|I.$$ 3

3. Answer any *two* of the following questions : 5×2=10

(a) *(i)* What is rank of a matrix? 2

(ii) Find the matrix A such that

$$A^{-1} = \begin{pmatrix} 3 & 2 \\ 1 & 6 \end{pmatrix}$$ 3

(b) *(i)* Prove that the product of a singular matrix with its adjoint is the null matrix. 2

(ii) Prove that inverse of the product of two square matrices is the same order as the product of their inverse in the reverse order. 3

(c) *(i)* Show that every square matrix can be expressed uniquely as the sum of a Hermitian and a skew-Hermitian matrix. 2

(ii) Prove that the trace of a product of two matrices is independent of the order of multiplication. 3

4. Answer any *two* of the following questions : 5×2=10

(a) Verify the Cayley–Hamilton theorem for the matrix A and compute the inverse of it, where

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ 5

(b) Find the adjoint and inverse of the matrix

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & -6 & -5 \\ 6 & -3 & 2 \end{pmatrix}$$ 5

(c) Find the transformation equations for a 90° rotation about the z-axis. 5

GROUP—B

(Electrostatics)

(*Marks* : 35)

5. Choose the correct answer : 1×3=3

(a) The force between two point charges in a dielectric medium is

(i) greater

(ii) smaller

(iii) same as that in vacuum

(b) Two identical conducting balls having positive charges q_1 and q_2 are separated by a distance r. If they are made in touch with each other and then separated to the same distance, the force between them will

(i) increase

(ii) decrease

(iii) remain unchanged

(iv) zero

(c) Which of the following is the equipotential surface?

(i) Informally charged disc

(ii) Charged spherical conductor

(iii) Circle with a point charge at the centre

(iv) Spherical surface with a point charge at the centre

6. Answer any *three* from the following questions : 2×3=6

(a) Write the Laplace's equation in cylindrical polar coordinates.

(b) "Reflection of lines of force and displacement is alike to light rays." Explain.

(c) What is molecular polarizability?

(d) What do you mean by electric images?

7. Answer any *two* of the following questions : 3×2=6

(a) Prove that the gravitational force is negligible in comparison to electrostatic force in the hydrogen atom in which the electron and proton are about $5 \cdot 3 \times 10^{-11}$ meter apart.

(b) A sphere of radius a is polarized along the radius vector. Find the volume and surface charge densities. Show that the total charge is zero.

(c) A spherical drop of water carrying a charge of 3×10^{-11} coulomb has a potential 500 volts at its surface. What is the radius of the drop? If two such drops of the same charge and radius combine to form a single drop, what is the potential at the surface of the new drop so formed?

8. Answer any *two* questions : 10×2=20

(a) (i) Calculate the potential at a point due to charge disc and show with diagram, the potential variation with distance. 5

(ii) A metal ball of radius 5 cm is given a charge of 100 e.s.u. Find the force acting on it per unit area. 5

(b) (i) Show that the induced surface charge on the dielectric in an induced field is always less than the free charge. What will be the induced charge when dielectric is not present? 4+1=5

(ii) Determine the electric field potential and electric field intensity when a point charge is above a ground conducting plane. 5

(c) (i) Show that in passing from empty space into a dielectric, the lines of force are bent away from the normal to the surface. 5

(ii) Solve Laplace equation to find the potential at a point at a distance r from the axis of an infinitely conducting cylinder of radius a_0 charged with a surface charge density σ. Take the potential of the cylinder to be zero. 5

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